The Magic of Mathematics: Discovering the Spell of Mathematics by Theoni Pappas (Wide Worlds Publishing/Tetra, 1994), ISBN 0-933174-99-3. Pages 133-137, & 235.

THE MATHEMATICALLY ANNOTATED GARDEN

"Good morning day!" exclaimed the gardener, as she greeted the sunrise and her plants. Little did she know that strange things were lurking in the leaves and rich soil. Deep in the roots of the plants were ***fractals and networks****, and from the cosmos, irises, marigolds, and daisies* ***Fibonacci numbers*** *were staring at her.*

She proceeded about her daily ritual of tending to her garden. At each place, something unusual appeared, but she was oblivious, captivated only by the obvious wonders that nature presented.

She first went to clear out her ferns. Removing the dead fronds to expose the new fiddle heads, she did not recognize the ***equiangular spirals*** *greeting her and the fractal-like formation of leaves on the ferns. Suddenly, as the breeze shifted, she was struck by the lovely fragrance of the honeysuckle. Looking over, she saw how it was taking over the fence and getting into the peas. She decided it definitely needed some judicious pruning. She did not realize that* ***helices*** *were at work, and the left-handed helices of the honeysuckle had wound around some of the right-handed helices of the peas. It required a careful hand to avoid damaging her new crop of peas.*

FRACTALS can appear as symmetrically changing/growing objects or as randomly asymmetrically changing objects. In either case, fractals are changing according to mathematical rules or patterns used to describe and dictate the growth of an initial object. Think of a geometric fractal as an endless generating pattern —the pattern continually replicates itself but in a smaller version. Thus, when a portion of a geometric fractal is magnified it looks exactly like the original version. In contrast, when a portion of a Euclidean object as a circle is magnified it begins to appear less curved. A fern is an ideal example of fractal replication. If you zero in on any portion of the fractal fern, it appears as the original fern leaf. A fractal fern can be created on a computer.

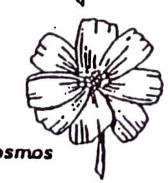
fractal tree

NETWORKS are mathematical diagrams which present a simpler picture of a problem or situation. Networks were used by Euler in the Königsberg bridge problem (see *The Spell of Logic, Recreation, & Games* section). He reduced the problem to a simple diagram, which he analyzed and solved. Today networks are tools used in topology.

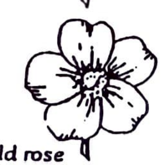
FIBONACCI NUMBERS 1, 1, 2, 3, 5, 8, 13, 21, ... Fibonacci (Leonardo da Pisa) was one of the leading mathematicians of the Middle Ages. Although he made significant contributions to the fields of arithmetic, algebra and geometry, he is popular today for this sequence of numbers, which happened to be the solution to an obscure problem appearing in his book Liber Abaci. In the 19th century, French mathematician Edouard Lucas edited a recreational mathematics work that included the problem. It was at this time that Fibonacci's name was attached to the sequence. In nature the sequence appears in:

bloodroot

trillum



cosmos



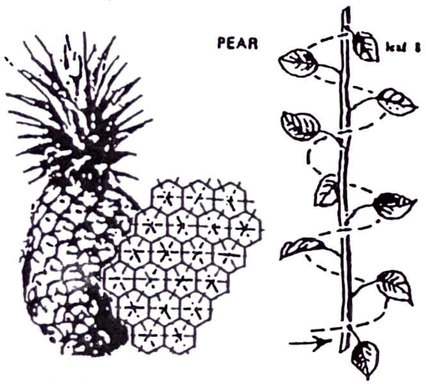
wild rose

Next she moved to weed beneath the palm tree she had planted to give her garden a somewhat exotic accent. Its branches were moving in the breeze, and she had no idea that ***involute curves*** *were brushing against her shoulders.*

She looked over at her corn smuggly. "Ha!" she thought. She had been hesitant to plant corn, but was encouraged by how well the young corn was progressing. Unbeknownst to her, ***triple junctions*** *of*

corn kernels would form within the ears.

How well the entire garden was shaping up and exploding with new growth! Admiring the new green leaves on the maple tree, she knew there was something inherently pleasing in their shape — nature's lines of ***symmetry*** *had done their work well.*



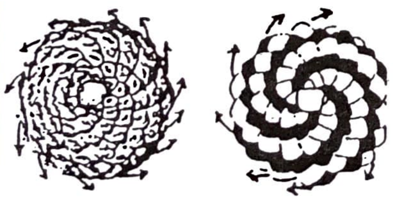
And nature's ***phyllotaxis*** *was only evident to the trained eye in budding leaves on branches and stems of plants.*

Glancing around, she focused on the carrot patch. She was proud of how they were doing, and noted they needed thinning to insure uniform good sized carrots. She did not want to rely on nature to ***tessellate*** *space with carrots.*

• *Flowers with a Fibonacci number of petals* (trilium, wildrose, bloodroot, cosmos, columbine, lily blossom, iris)

• *Arrangement of leaves, twigs and stems* is known as **phyllotaxis.**

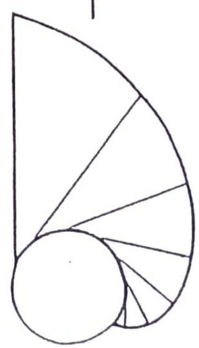
Select a leaf on a stem and count the number of leaves (assuming none have been broken-off) until you reach one directly in line with the one you selected. The total number of leaves (not counting the first one you selected) is usually a Fibonacci number in many plants, such as in elm, cherry or pear trees.

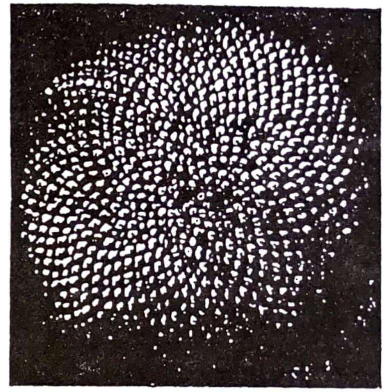


• *The pine cone numbers:* If the left and right handed spirals on a pine cone are counted, the two numbers are very often consecutive Fibonacci numbers. This also holds true for sunflower seedheads and seedheads of other flowers. The same is true of pineapples. Looking at the base of a pineapple count the number of left and right spirals composed of hexagonal shapes scales. They should be consecutive Fibonacci numbers.

SPIRALS & HELICES:
Spirals are mathematical forms which appear in many facets of nature, such as the curve of a fiddlehead fern, vines, shells, tornadoes, hurricanes, pine cones, the Milky Way, whirlpools. There are flat spirals, three dimensional spirals, right and left handed spirals, equiangular, logarithmic, hyperbolic, Archimedean spirals, and helices are just some of the many types of spirals which mathematics describes. The equiangular spiral appears in such growth forms of nature as the nautilus shell, a sunflower seedhead, the webs of Orb spiders. Some of the properties of the equiangular spiral are—angles formed from tangents to the spiral's radii are congruent (hence the term equiangular) — it increases at a geometric rate, thereby any radius is cut by the spiral into sections that form a geometric progression — its shape remains the same as it grows.

INVOLUTE CURVE: As a rope is wound or unwound around another curve (here a circle), it describe an involute curve. Involute is the shape found in the beak of an eagle, the dorsal fin of a shark, and the tip of a hanging palm leaf.

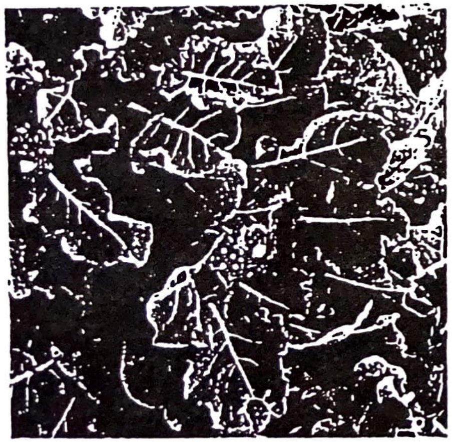




She had no idea that the garden abounded with ***equiangular spirals.*** *They were in the seedheads of the daisies and various flowers. Many things that grow form this spiral because of how it retains its shape while its size increases.*

It was getting warm, so she decided she would continue the cultivation when the sun shifted. Meanwhile, she made one final assessment — admiring the combination of flowers, vegetables and other plants she had so thoughtfully selected. But once more something escaped her. Her garden was full of spheres, cones, polyhedra and other geomet *shapes, and she did not recognize them.*

As nature puts forth its wonders in the garden, most people are oblivious to the massive calculations and mathematical work that have become so routine in nature. Nature knows well how to work with restrictions of material and space, and produce the



Many types of symmetry appear in the garden. For example, in the above photograph one can find point symmetry in broccoli florettes and line symmetry in their leaves.

most harmonious forms. And so, during each day of spring, the gardener will enter her domain with a gleam in her eye. She will seek out the new growth and blossoms each day brings, unaware of the mathematical beauties flowering in her yard.

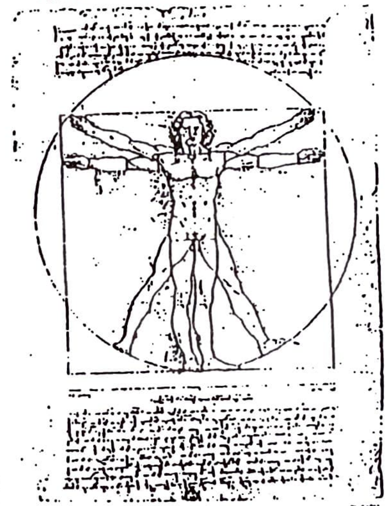
TRIPLE JUNCTION: A triple junction is the point where three line segments meet, and the angles at the intersection are each to 120°. Many natural occurrences result from restrictions caused by boundaries or availability of space. Triple junction is an equilibrium point toward which certain natural occurrences tend. Among other things, it is found in soap bubble clusters, the formation of kernels on the cob of corn, and the cracking of earth or stone.

SYMMETRY: Symmetry is that perfect balance one sees and senses in the body of a butterfly, in the shape of a leaf, in the form of the human body, in the perfection of a circle. From a mathematical point of view, an object is considered to possess line symmetry if one can find a line which divides it into two identical parts so that if it were possible to fold along that line both parts would match perfectly over one another. An object has point symmetry if infinitely many such lines exist for a particular point, for example a circle has point symmetry with respect to its center point.

TESSELLATE: To tessellate a plane simply means being able to cover the plane with flat tiles so tha there are no gaps and no tiles overlap, such as with regular hexagons, squares, or other objects. Space is tessellated or filled by three-dimensional objects such as cubes, or truncated octahedra.

SECRETS OF TH
RENAISSANCE MA

This famous drawing by Leonardo da Vinci appeared in the book, *De Divina Proportione,* which Leonardo illustrated for mathematician Luca Paoli in 1509.



Leonardo wrote an extensive section on the proportions of the human body in one of his notebooks. He determined measurements and proportions for all parts of the body, including the head, eyes, ears, hands and feet. His proportions were based on numerous studies, observations and measurements. In his notebook, he also made reference to the works of Vitruvius, the Roman architect (circa 30 B.C.) who also dealt with the proportions of the human body. Leonardo writes of how he was influenced by Vitruvius:

> *Vitruvius, the architect, says in his works on architecture that the measurements of the human body are distributed by Nature as follows: ...If you open your legs so much as to decease your height by 1/14 and spread and raise your arms till your middle fingers touch the level of the top of your head you must know that the center of the outspread limbs will lie in the navel and the space between the legs will be an equilateral triangle.*

Leonardo adds, *The length of a man's outspread arms is equal to his height.*•[1]

[1]Richter, Jean Paul, editor. *The Notebooks of Leonardo da Vinci, vol. I.* Dover Publications, 1970, New York.